

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

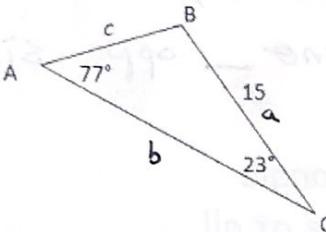
6.1/6.2 Law of Sine and Law of Cosine

*Law of Sine and Law of Cosine worksheets

Sine: must know 2 \angle s & 1 side AAS or ASA

Examples: Round to four decimal places.

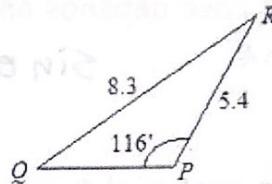
- 1) Find c .
- 2) Find the measure of $\angle Q$.



$$\frac{c}{\sin 23} = \frac{15}{\sin 77}$$

$$c = \frac{15 \sin 23}{\sin 77}$$

$$c = 6.0151$$



$$\frac{8.3}{\sin 116} = \frac{5.4}{\sin Q}$$

$$\sin Q = \frac{5.4 \sin 116}{8.3}$$

$$Q = \sin^{-1}\left(\frac{5.4 \sin 116}{8.3}\right)$$

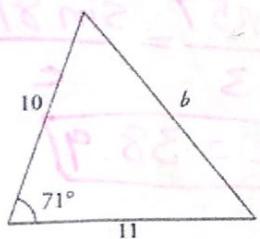
$$Q = 35.7859^\circ$$

$$Q = 36^\circ \text{ or }$$

$$180 - 36 \\ = 144^\circ \\ \text{Too big}$$

Examples: Round to four decimal places.

- 3) Find b .
- 4) Find $m\angle A$.

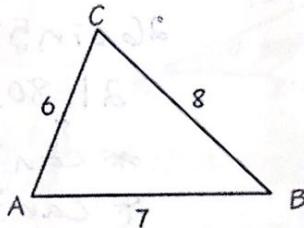


$$b^2 = 10^2 + 11^2 - 2(10)(11) \cos 71$$

$$b^2 = 221 - 220 \cos 71$$

$$\sqrt{b^2} = \sqrt{149.375006}$$

$$b = 12.2219$$



$$8^2 = 6^2 + 7^2 - 2(6)(7) \cos A$$

$$64 = 85 - 84 \cos A$$

$$-21 = -84 \cos A$$

$$\cos^{-1} \frac{1}{4} = \cos A \cdot \cos^{-1}$$

$$A > 75.5225^\circ$$

6.1/6.2 The Ambiguous Case and Applications

Ambiguous Case of the Law of Sines (SSA)

*The given information may result in one triangle, two triangles, or no triangle at all

*The number of possible triangles, if any, that can be formed in the SSA case depends on h (the length of the altitude) where $h = b \sin A$

$$\frac{\sin \theta}{\text{hyp}} = \frac{\text{opp}}{\sin \text{Test}}$$

Case 1) a is just right, we have one right triangle

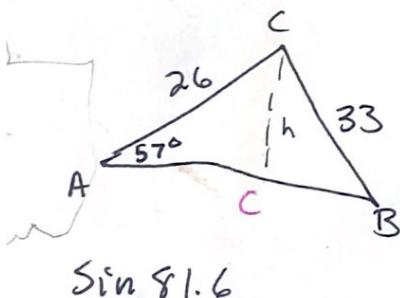
Case 2) a is too small, we don't have a triangle at all

Case 3) a is too big

- and a is bigger than b , we have one triangle (pendulum away from A)
- and a is smaller than b , we have two triangles (pendulum in and out from A)

One Solution:

1) Solve triangle ABC



$\sin 81.6$



$a > h$
 $a < b$
swing 2 ways

$A = 57^\circ$ $a = 33$ $\frac{\sin \text{Test}}{\text{opp}}$

$b = 26$ $\frac{\text{side}}{\text{opp}}$

$$26 \sin 57 = 33$$

$$21.805 < 33$$

- * can't be right
- * can't be 2 Δs

$$\frac{\sin 57}{33} = \frac{\sin 81.6}{c}$$

$$c = 38.9$$

$$\frac{\sin 57}{33} = \frac{\sin B}{26}$$

$$\frac{26 \sin 57}{33} = \sin B$$

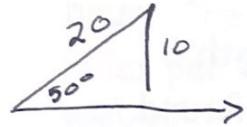
$$B = \sin^{-1} \left(\frac{26 \sin 57}{33} \right)$$

$$\boxed{B = 41.4^\circ}$$

$$\begin{aligned} \angle C &= 180 - 57 - 41.4 \\ \angle C &= 81.6^\circ \end{aligned}$$

No Solution:

2) Solve triangle ABC $A = 50^\circ$ $a = 10$ $b = 20$



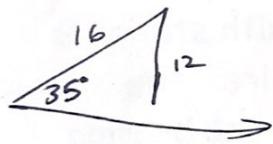
$$20 \sin 50^\circ > 10$$

$$15.32 > 10$$

No Δ since $15.32 > 10$

Two Solutions:

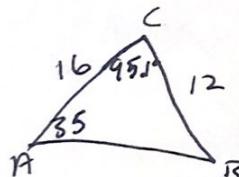
3) Solve triangle ABC $A = 35^\circ$ $a = 12$ $b = 16$



$$16 \sin 35^\circ > 12$$

$$9.17 < 12$$

$$\therefore 12 < 16$$



$$\frac{\sin 35}{12} = \frac{\sin B}{16}$$

$$\sin B = 16 \sin 35$$

$$B = \sin^{-1} \left(\frac{16 \sin 35}{12} \right)$$

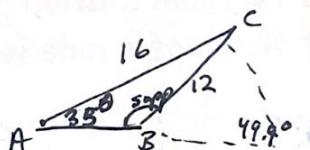
$$\angle B = 49.9^\circ$$

$$\angle C = 180 - 35 - 49.9 = 95.1^\circ$$

$$\frac{\sin 35}{12} = \frac{\sin 95.1}{c}$$

$$c = \frac{12 \sin 95.1}{\sin 35}$$

$$c = 20.8$$



$$\angle B = 180 - 49.9 = 130.1^\circ$$

$$\angle C = 180 - 35 - 130.1 = 14.9^\circ$$

$$\frac{\sin 35}{12} = \frac{\sin 14.9}{c}$$

$$c = \frac{12 \sin 14.9}{\sin 35}$$

$$c = 5.379$$

Area of an oblique Triangle

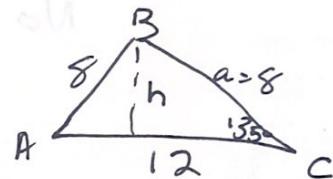
$$\begin{aligned} \text{Area} &= \frac{1}{2} bc \sin A \\ &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} ac \sin B \end{aligned} \quad \left. \begin{array}{l} \text{included} \\ \angle s \end{array} \right\}$$

- 4) Find the area of a triangle having two sides of length 8 meters and 12 meters and an included angle of 135° . Round to the nearest square meter.

$$A = \frac{1}{2}(8)(12) \sin 135$$

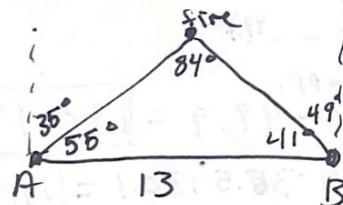
$$A = 33.94$$

$$\boxed{A = 34 \text{ m}^2}$$



Applications

- 5) Two fire-lookout stations are 13 miles apart, with station B directly east of station A. Both stations spot a fire. The bearing of the fire from station A is N 35° E and the bearing of the fire from station B is N 49° W. How far, to the nearest tenth of a mile is the fire from station B?



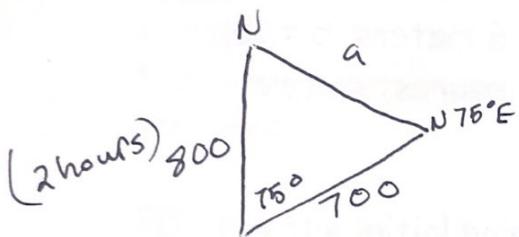
$$\angle C = 180 - 55 - 41 = 84^\circ$$

$$\frac{\sin 84}{13} = \frac{\sin 55}{4}$$

$$a = \frac{13 \sin 55}{\sin 84} = 10.7$$

The fire is approximately
10.7 miles from station
B.

6) Two airplanes leave an airport at the same time on different runways. One flies directly north at 400 miles per hour. The other airplane flies on a bearing of N 75° E at 350 miles per hour. How far apart will the airplanes be after two hours?



$$a^2 = 800^2 + 700^2 - 2(800)(700)\cos 75$$

$$a^2 = 1130000 - 1120000 \cos 75$$

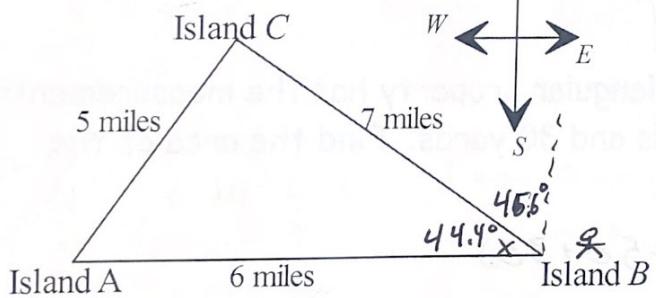
$$\sqrt{a^2} = \sqrt{840122.6695}$$

$$a = 916.58 \approx 917 \text{ miles apart.}$$

Use law of cosine

Since SAS
included
angle

7)



If you are on island B, what bearing should you navigate to go to island C?

$$5^2 = 6^2 + 7^2 - 2(6)(7)\cos x$$

$$25 = 85 - 84 \cos x$$

$$-60 = -84 \cos x$$

$$\cos^{-1} 0.714 = \cos x \cdot \cos^{-1}$$

$$x = 44.4153 \approx 44.4^\circ$$

$$90 - 44.4 = 45.6^\circ$$

N 45.6° W

Heron's Formula:

The area of a triangle with sides a , b and c is

$$\sqrt{s(s-a)(s-b)(s-c)}$$

where s is one-half its perimeter: $s = \frac{1}{2}(a+b+c)$

- 8) Find the area of the triangle with $a = 6$ meters, $b = 16$ meters, and $c = 18$ meters. Round to the nearest square meters.

$$s = \frac{1}{2}(6+16+18)$$

$$s = 20$$

$$A = \sqrt{20(20-6)(20-16)(20-18)}$$

$$A = \sqrt{20(14)(4)(2)}$$

$$A = \sqrt{2240}$$

$$\boxed{\approx 47 \text{ m}^2}$$

- 9) Bob Fernando's triangular property has the measurements of 40 yards, 50 yards and 30 yards. Find the area of the property.

$$s = \frac{1}{2}(40+50+30)$$

$$s = 60$$

$$A = \sqrt{60(60-40)(60-50)(60-30)}$$

$$A = \sqrt{60(20)(10)(30)}$$

$$A = \sqrt{360000}$$

$$A = 600 \text{ yd}^2$$

6.6 Vectors

A vector is a directed line segment

- Magnitude (length) $\|m\|$ - distance of m $= \sqrt{x^2 + y^2}$

- Direction (angle measure) degrees or radians $= \tan^{-1} \frac{y}{x}$

*For vectors to be equal, they must have the same magnitude and direction.

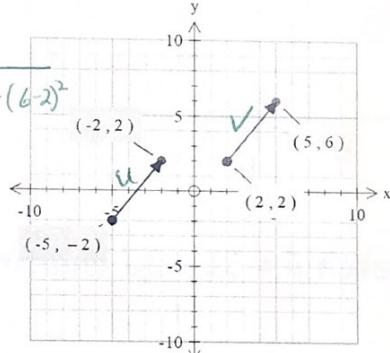
*Vectors are usually denoted by boldface letters, but can also be written as \vec{v}

\overrightarrow{PQ} P is the initial point and Q is the terminal point

1) show that $\mathbf{u} = \mathbf{v}$

$$\begin{aligned}\|\mathbf{u}\| &= \sqrt{(-2+5)^2 + (2+2)^2} \\ &= \sqrt{9+16} \\ &= \sqrt{25} \\ &= 5\end{aligned}$$

$$\begin{aligned}\|\mathbf{v}\| &= \sqrt{(5-2)^2 + (6-2)^2} \\ &= \sqrt{9+16} \\ &= \sqrt{25} \\ &= 5\end{aligned}$$



Direction

$$\begin{aligned}u: m &= \frac{2+2}{-2+5} \\ &= \frac{4}{3}\end{aligned}$$

$$\begin{aligned}v: m &= \frac{6-2}{5-2} \\ &= \frac{4}{3}\end{aligned}$$

They have the same magnitude
in direction, $\therefore \|\vec{v}\| = \|\vec{u}\|$

Scalar Multiplication

If k is a real number and \mathbf{v} a vector, the vector $k\mathbf{v}$ is called a scalar multiple of the vector \mathbf{v} .

- Magnitude of $|k|\|\mathbf{v}\|$
- Direction
 - o Same if $k > 0$
 - o Opposite if $k < 0$

Adding Vectors

- Add the x's
- Add the y's

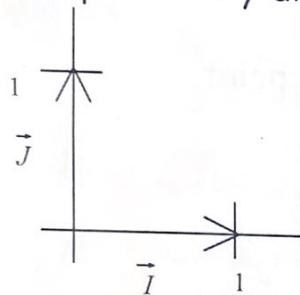
Subtracting Vectors

- Subtract the x's
- Subtract the y's

Unit Vectors

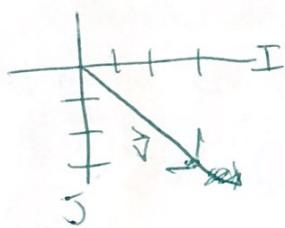
I represents x distance (horizontal component)

J represents y distance (vertical component)



2) Sketch $\vec{v} = 3I - 3J$ and find $\|\vec{v}\|$

Starts @ $(0, 0)$ & ends @ $(3, -3)$



$$\begin{aligned}a^2 + b^2 &= c^2 \\3^2 + (-3)^2 &= c^2 \\9 + 9 &= c^2 \\ \vec{v} &= 3\sqrt{2}\end{aligned}$$

3) If $\vec{v} = 7I + 3J$ and $\vec{w} = 4I - 5J$, find $\vec{v} + \vec{w}$ and $\vec{v} - \vec{w}$



$$\vec{v} + \vec{w} = (7I + 3J) + (4I - 5J) = 11I - 2J$$

$$\vec{v} - \vec{w} = (7I + 3J) - (4I - 5J) = 3I + 8J$$

4) If $\vec{v} = 7\mathbf{i} + 10\mathbf{j}$, find $8\vec{v}$ and $-5\vec{v}$

$$8\vec{v} = 8(7\mathbf{i} + 10\mathbf{j}) = 56\mathbf{i} + 80\mathbf{j}$$

$$-5\vec{v} = -5(7\mathbf{i} + 10\mathbf{j}) = -35\mathbf{i} - 50\mathbf{j}$$

5) If $\mathbf{v} = 7\mathbf{i} + 3\mathbf{j}$ and $\mathbf{w} = 4\mathbf{i} - 5\mathbf{j}$, find $6\mathbf{v} - 3\mathbf{w}$

$$6\mathbf{v} = 6(7\mathbf{i} + 3\mathbf{j}) = 42\mathbf{i} + 18\mathbf{j}$$

$$3\mathbf{w} = 3(4\mathbf{i} - 5\mathbf{j}) = \underline{-12\mathbf{i} + 15\mathbf{j}}$$
$$\boxed{30\mathbf{i} + 33\mathbf{j}}$$

Zero Vector: $\vec{v} = 0\mathbf{i} + 0\mathbf{j}$

$$\|\vec{v}\| = 0$$

Direction = none

Unit vector in the same direction:

- vector • Unit vector has a magnitude of 1 therefore we divide the
whose mag. = 1 vector by its own magnitude.

$$\frac{\sqrt{}}{\|\mathbf{v}\|}$$

6) Find the unit vector in the same direction of $\vec{v} = 4\mathbf{i} - 3\mathbf{j}$.

$$\|\vec{v}\| = \sqrt{a^2 + b^2} = \sqrt{4^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$$

$$\frac{\vec{v}}{\|\vec{v}\|} = \frac{4\mathbf{i} - 3\mathbf{j}}{5} = \boxed{\frac{4}{5}\mathbf{i} - \frac{3}{5}\mathbf{j}}$$

Verify
has to be 1

$$\sqrt{\left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2} = \sqrt{\frac{16}{25} + \frac{9}{25}} = \sqrt{\frac{25}{25}} = \sqrt{1} = 1 \quad \checkmark$$

Writing a vector in terms of its magnitude and direction:

$$\begin{aligned} \vec{v} &= \underbrace{\|\vec{v}\| \cos \theta \cdot \mathbf{i}}_{x} + \underbrace{\|\vec{v}\| \sin \theta \cdot \mathbf{j}}_{y} \\ \cos \theta &= \frac{x}{\|\vec{v}\|} \quad a = \|\vec{v}\| \cos \theta \\ \sin \theta &= \frac{y}{\|\vec{v}\|} \quad b = \|\vec{v}\| \sin \theta \\ \vec{v} &= ai + bj \end{aligned}$$

*A vector that represents the direction and speed of an object in motion is called a **velocity vector**.

7) The jet stream is blowing at 60 miles per hour in the direction N 45° E. Express its velocity as a vector \vec{v} in terms of \mathbf{i} and \mathbf{j} .

$$\|\vec{v}\| = 60$$



$$\vec{v} = \|\vec{v}\| \cos \theta \mathbf{i} + \|\vec{v}\| \sin \theta \mathbf{j}$$

$$= 60 \cos 45^\circ \mathbf{i} + 60 \sin 45^\circ \mathbf{j}$$

$$= 60 \left(\frac{\sqrt{2}}{2}\right) \mathbf{i} + 60 \left(\frac{\sqrt{2}}{2}\right) \mathbf{j}$$

$$\boxed{\vec{v} = 30\sqrt{2}\mathbf{i} + 30\sqrt{2}\mathbf{j}}$$

Finding the Resultant Force:

- 8) Two forces, \mathbf{F}_1 and \mathbf{F}_2 , of magnitude 30 and 60 pounds, respectively, act on an object. The direction of \mathbf{F}_1 is N 10° E and the direction of \mathbf{F}_2 is N 60° E. Find the magnitude, to the nearest hundredth of a pound, and the direction angle, to the nearest tenth of a degree, of the resultant force.

$$\mathbf{F}_1 \text{ 30 N } 10^\circ \text{ E} \quad \|\mathbf{F}_1\| = 30 \quad \theta = 90 - 10 = 80^\circ$$

$$\mathbf{F}_2 \text{ 60 N } 60^\circ \text{ E} \quad \|\mathbf{F}_2\| = 60 \quad \theta = 90 - 60 = 30^\circ$$

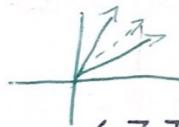
$$\begin{aligned} \mathbf{F}_1 &= 30 \cos 80^\circ \mathbf{i} + 30 \sin 80^\circ \mathbf{j} & \mathbf{F}_2 &= 60 \cos 30^\circ \mathbf{i} + 60 \sin 30^\circ \mathbf{j} \\ &= 5.209 \mathbf{i} + 29.544 \mathbf{j} & &= 51.962 \mathbf{i} + 30 \mathbf{j} \end{aligned}$$

Resultant Force

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = 57.171 \mathbf{i} + 59.544 \mathbf{j}$$

$$\|\mathbf{F}\| = \sqrt{(57.171)^2 + (59.544)^2}$$

$$= \sqrt{6814.011} \approx 82.547$$



magnitude of
the resultant
force (lbs.)

equivalent
to the 2
Forces.

$$\tan^{-1} \left(\frac{59.544}{57.171} \right)$$

$$= 46.16^\circ$$

$$\cancel{-30^\circ}$$

$$\cancel{16.16^\circ}$$

$$\theta = 16.16^\circ$$

?

6.7 The Dot Product

The dot product of two vectors is the sum of the products of their horizontal components and their vertical components.

$$\text{If } \vec{v} = a_1 \mathbf{i} + b_1 \mathbf{j} \quad \text{and} \quad \vec{w} = a_2 \mathbf{i} + b_2 \mathbf{j}$$

$$\text{Then } \vec{v} \cdot \vec{w} = a_1 a_2 + b_1 b_2$$

*This gives us a number, not a vector

*Two vectors are orthogonal if the angle between them is 90°

1) If $\vec{v} = 7\mathbf{i} - 4\mathbf{j}$ and $\vec{w} = 2\mathbf{i} - \mathbf{j}$, find each of the following dot products:

$$\vec{v} \cdot \vec{w}$$

$$= 7(2) + (-4)(-1)$$

$$= 14 + 4$$

$$= \boxed{18}$$

$$\vec{w} \cdot \vec{v}$$

$$\boxed{18}$$

$$\vec{w} \cdot \vec{w}$$

$$(2)(2) + (-1)(-1)$$

$$4 + 1$$

$$\boxed{5}$$

Properties of the Dot Product

~~1) $\vec{v} \cdot \vec{v} = \vec{v} \cdot \vec{v}$~~

~~2) $\vec{v}\vec{v} \vec{v}\vec{v} \vec{v}\vec{v} \vec{v}$~~

~~3) \vec{v}~~

~~4) $\vec{v}\vec{v} \vec{v}$~~

~~5) $\vec{v}\vec{v} \vec{v}\vec{v} \vec{v}\vec{v}$~~

If u, v, w are vectors,
 c is scalar

$$v \cdot u = u \cdot v$$

$$u(v+w) = uv+uw$$

$$0 \cdot v = 0$$

$$v \cdot v = \|v\|^2$$

$$(cu)\vec{v} = c(u \cdot v) = u(cv)$$

Formula for the angle between two vectors:

If v and w are two nonzero vectors and θ is the smallest nonnegative angle between v and w , then

$$\cos \theta = \frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\| \|\vec{w}\|} \quad \text{and} \quad \theta = \cos^{-1} \left(\frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\| \|\vec{w}\|} \right)$$

- 2) Find the angle between the vectors $v = 4i - 3j$ and $w = i + 2j$

Round to the nearest tenth of a degree.

$$\|v\| = \sqrt{4^2 + 3^2} = 5$$

$$\|w\| = \sqrt{1^2 + 2^2} = \sqrt{5}$$

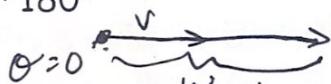
$$\cos \theta = \frac{4(1) + (-3)(2)}{5(\sqrt{5})}$$

$$= \frac{4 - 6}{5\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$$

$$\cos^{-1} \cos \theta = \frac{\cos^{-1} \frac{-2\sqrt{5}}{25}}{25} \quad \theta = 100.3^\circ$$

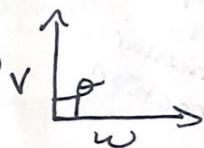
Parallel and Orthogonal Vectors:

- Two vectors are parallel when the angle between the vectors is 0° or 180°



- Two vectors are orthogonal when the angle between the vectors is 90°

*The word orthogonal is used rather than perpendicular to describe vectors that meet at right angles.



- If the dot product is 0, the vectors are orthogonal

$\vec{v} \cdot \vec{w} = 0$, Then $\vec{v} \perp \vec{w}$ are orthogonal (Opp reciprocal slope)

3) Are the vectors $\vec{v} = 2i + 3j$ and $\vec{w} = 6i - 4j$ orthogonal?

$$\begin{aligned}\vec{v} \cdot \vec{w} &= 2(6) + (3)(-4) \\ &= 12 - 12 \\ &= 0\end{aligned}$$

Since $\vec{v} \cdot \vec{w} = 0$, then the vectors are
orthogonal.

The Vector Projection of v onto w :

If v and w are two nonzero vectors, the vector projection of v onto w is:

a vector
as the sum
of 2 orthogonal
vectors.

$$V_1 = \text{Proj}_w v = \frac{\vec{v} \cdot \vec{w}}{\|\vec{w}\|^2} \cdot \vec{w}, \quad V_2 = v - v_1$$

This tells us how much v is parallel to w and v_2 is orthogonal to w .

The vectors v_1 and v_2 are called the **vector components** of v .

in a particular direction

The process of expressing v as $v_1 + v_2$ is called the **decomposition** of v into v_1 and v_2

- 4) If $v = 2i - 5j$ and $w = i - j$, find the vector projection of v onto w .

$$\begin{aligned} \text{Proj}_w v &= \frac{v \cdot w}{\|w\|^2} w = \frac{2(1) + (-5)(-1)}{(\sqrt{2})^2} \cdot (i - j) \\ &= \frac{7}{2}(i - j) = \boxed{\frac{7}{2}i - \frac{7}{2}j} \end{aligned}$$

- 5) Let $v = 2i - 5j$ and $w = i - j$. Decompose v into two vectors v_1 and v_2 where v_1 is parallel to w and v_2 is orthogonal to w .

$$v_1 = \text{Proj}_w v = \frac{7}{2}i - \frac{7}{2}j \quad (\text{from previous example})$$

$$\begin{aligned} v_2 &= v - v_1 = (2i - 5j) - \left(\frac{7}{2}i - \frac{7}{2}j\right) \\ &= \frac{4}{2}i - \frac{10}{2}j - \frac{7}{2}i + \frac{7}{2}j \end{aligned}$$

$$\boxed{v_2 = -\frac{3}{2}i - \frac{3}{2}j}$$

- 6) A child pulls a wagon along level ground by exerting a force of 20 lbs on a handle that makes an angle of 30° with the horizontal. How much work is done pulling the wagon 150 ft?

$$\text{Def. of work: } w = \vec{F} \cdot \vec{AB} = \|F\| \|AB\| \cos \theta$$

mag of force distance over which constant force is applied \angle between force & direction of motion

$$(20)(150)\cos 30$$

$$= 2598.07$$

\approx 2598 foot pounds

6.3 Polar Coordinates

* Many times it is much easier to graph on a polar coordinate plane rather than rectangular coordinate plane.

Talk about $r = 3$ & $\theta = \frac{\pi}{4}$

A point P on the polar coordinate = (r, θ) .

- R is a directed distance from the pole to P (can be pos or neg)
- θ is an angle from the polar axis to the line segment from the pole to P (can be degrees or radians)
 - Positive angles are measured counterclockwise
 - Negative angles are measured clockwise

Relations between polar and rectangular coordinates
(conversions)

$$x = r \cos\theta$$

$$r = \sqrt{x^2 + y^2}$$

$$y = r \sin\theta$$

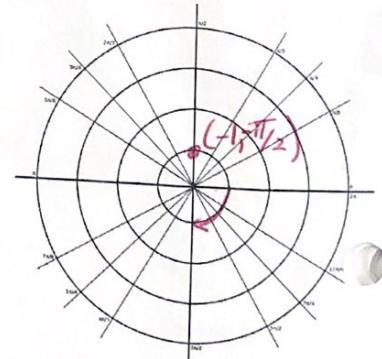
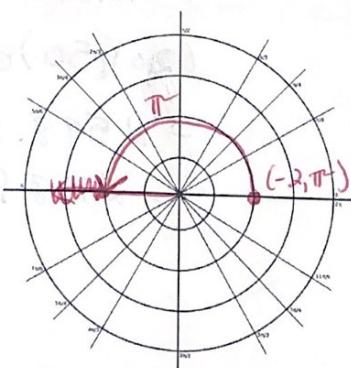
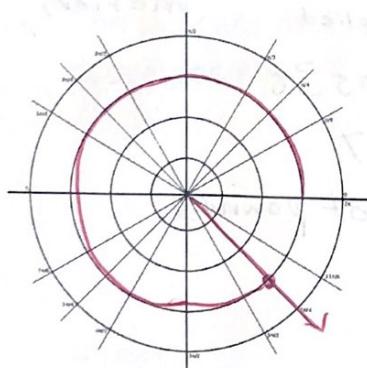
$$\theta = \tan^{-1} \left(\frac{y}{x} \right)$$

1) Plot the following polar coordinates:

a. $(3, 315^\circ)$

b. $(-2, \pi)$

c. $(-1, -\frac{\pi}{2})$



2) Find another representation of $(5, \frac{\pi}{4})$ in which:

r is positive and $2\pi < \theta < 4\pi$

$$+2\pi \quad (5, \frac{9\pi}{4})$$

add 2π & don't change r

r is negative and $0 < \theta < 2\pi$

Add π & replace r by $-r$

$$(-5, \frac{5\pi}{4})$$

r is positive and $-2\pi < \theta < 0$

subtract 2π & do not change r

$$(5, -\frac{7\pi}{4})$$

3) Find the rectangular coordinates of the points with the following polar coordinates:

a. $(3, \pi)$

$$\begin{aligned} x &= r \cos \theta = 3 \cos \pi \\ &= 3(-1) \\ &= -3 \\ y &= r \sin \theta = 3 \sin \pi \\ &= 3(0) \\ &= 0 \end{aligned}$$

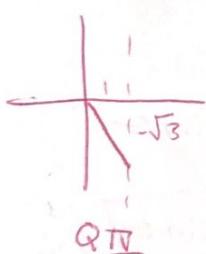
$$[-3, 0]$$

b. $(-10, \frac{\pi}{6})$

$$\begin{aligned} x &= -10 \cos \frac{\pi}{6} & y &= -10 \sin \frac{\pi}{6} \\ &= -10(\frac{\sqrt{3}}{2}) & &= -10(\frac{1}{2}) \\ &= -5\sqrt{3} & &= -5 \end{aligned}$$

$$[-5\sqrt{3}, -5]$$

4) Find polar coordinates of the point whose rectangular coordinates are $(1, -\sqrt{3})$.



$$r = \sqrt{1^2 + (-\sqrt{3})^2}$$

$$= \sqrt{4}$$

$$= 2$$

$$\tan \theta = \frac{-\sqrt{3}}{1}$$

$$\theta = \tan^{-1} -\sqrt{3}$$

$$\theta = \frac{5\pi}{3}$$

$$\boxed{(2, \frac{5\pi}{3})}$$

5) Find polar coordinates of the point whose rectangular coordinates are $(0, -4)$. Express θ in radians.

$$\begin{aligned}
 r &= \sqrt{0^2 + 4^2} \\
 &= \sqrt{16} \\
 &= 4
 \end{aligned}
 \quad
 \begin{aligned}
 \tan \theta &= \frac{-4}{0} \\
 &\text{undefined} \\
 \theta &= \frac{3\pi}{2}
 \end{aligned}$$

$$\boxed{(4, \frac{3\pi}{2})}$$

6) Convert each rectangular equation to a polar equation that expresses r in terms of θ :

$$\begin{aligned}
 x &= r\cos\theta \\
 y &= r\sin\theta
 \end{aligned}
 \quad
 \text{a. } 3x - y = 6$$

$$\begin{aligned}
 3r\cos\theta - r\sin\theta &= 6 \\
 r(3\cos\theta - \sin\theta) &= 6
 \end{aligned}$$

$$\boxed{r = \frac{6}{3\cos\theta - \sin\theta}}$$

Standard form for circle

$$b. x^2 + (y + 1)^2 = 1 \quad r = 1 \text{ center}(0, -1)$$

$$\begin{aligned}
 (r\cos\theta)^2 + (r\sin\theta + 1)^2 &= 1 \\
 r^2 \cos^2\theta + r^2 \sin^2\theta + 2r\sin\theta + 1 &= 1 \\
 r^2(\cos^2\theta + \sin^2\theta) + 2r\sin\theta &= 0
 \end{aligned}$$

$$r^2 + 2r\sin\theta = 0$$

$$r(r + 2\sin\theta) = 0$$

$$r = 0 \quad \text{or} \quad r = -2\sin\theta$$

Single point
(the pole)

$$\boxed{r = -2\sin\theta}$$

Polar \rightarrow Rectangular $x^{\frac{1}{2}}y$ instead of $r^{\frac{1}{2}}\theta$

$$r^2 = x^2 + y^2 \quad r\cos\theta = x \quad r\sin\theta = y \quad \tan\theta = \frac{y}{x}$$

7) Convert each polar equation to a rectangular equation in x and y.

a. $r = 4$

$$\begin{aligned} r^2 &= 16 \\ x^2 + y^2 &= 16 \end{aligned}$$

b. $\theta = \frac{3\pi}{4}$

$$\begin{aligned} \tan\theta &= \frac{y}{x} \\ \tan\theta &= \tan\frac{3\pi}{4} \end{aligned}$$

(take tan of each side)

$$\tan\theta = -1$$

$$\frac{y}{x} = -1 \quad \rightarrow \quad y = -x$$

c. $r = -2\sec\theta$

$$\begin{aligned} r\cos\theta &= x & \sec = \frac{1}{\cos} \\ r &= \frac{-2}{\cos\theta} \end{aligned}$$

$$r\cos\theta = -2$$

$$x = -2$$

d. $r = 10\sin\theta$ mult. both sides by r

$$r^2 = 10r\sin\theta$$

$$x^2 + y^2 = 10y$$

$$x^2 + y^2 - 10y = 0 \quad \text{complete the square}$$

$$x^2 + y^2 - 10y + 25 = 25$$

$$x^2 + (y-5)^2 = 25$$

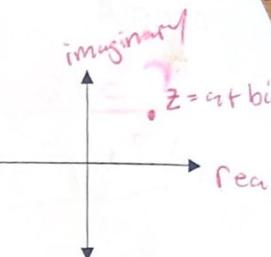
6.4 Graphs of Polar Coordinates

*Handout

6.5 Complex Numbers in Polar Form

Complex Form: $a + bi$ (rectangular form)

Real Imaginary



The absolute value of a complex number:

*Distance from the origin to the point z in the complex plane

$$|z| = |a + bi| = \sqrt{a^2 + b^2} = r$$

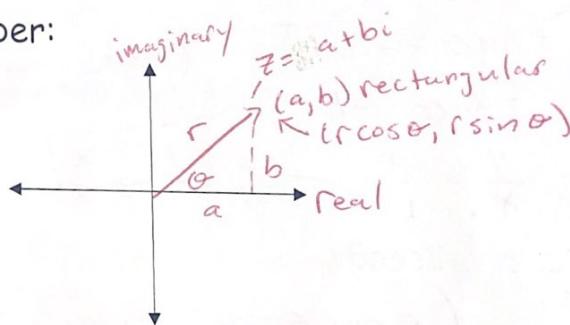
Polar Form of a complex number:

$$r = \sqrt{a^2 + b^2}$$

$$a = r \cos \theta$$

$$b = r \sin \theta$$

$$\theta = \tan^{-1} \frac{b}{a}$$



$$z = r \cos \theta + (r \sin \theta)i$$

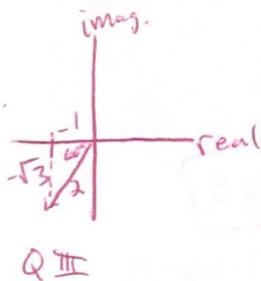
or

$$z = r(\cos \theta + i \sin \theta)$$

~~Meristet~~

~~r cis theta~~

- 1) Plot $z = -1 - i\sqrt{3}$ in the complex plane. Then write z in polar form. Express the argument in radians.



$$r = \sqrt{(-1)^2 + (-\sqrt{3})^2}$$

$$r = \sqrt{1+3}$$

$$r = \sqrt{4}$$

$$r = 2$$

$$\tan \theta = \frac{-\sqrt{3}}{-1}$$

$$= \sqrt{3}$$

$$= 4\pi/3$$

$$z = r(\cos \theta + i \sin \theta)$$

$$z = 2(\cos 4\pi/3 + i \sin 4\pi/3)$$

$$= 2(\cos 4\pi/3 + i \sin 4\pi/3)$$

$$2 \text{cis } \frac{4\pi}{3}$$

2) Write $z = 4(\cos 30^\circ + i \sin 30^\circ)$ in rectangular form.

$$= 4\left(\frac{\sqrt{3}}{2} + i\left(\frac{1}{2}\right)\right)$$

$$\boxed{z = 2\sqrt{3} + 2i}$$

$$z = a + bi$$

Product of Two Complex Numbers in Polar Form:

-Product/Multiply

Multiply the radii and add the θ 's

-Divide/Quotient

Divide the radii and subtract the θ 's

3) Find the product of $z_1 = 6(\cos 40^\circ + i \sin 40^\circ)$ and $z_2 = 5(\cos 20^\circ + i \sin 20^\circ)$. Leave the answer in polar form.

$$[6(\cos 40^\circ + i \sin 40^\circ)][5(\cos 20^\circ + i \sin 20^\circ)]$$

$$(6 \cdot 5)[\cos(40+20) + i \sin(40+20)]$$

$$\boxed{= 30(\cos 60^\circ + i \sin 60^\circ)} = 30 \text{ c is } 60^\circ \text{ or } 30 \text{ c is } \pi/3$$

4) Find the quotient of $z_1 = 50(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3})$ and

$$z_2 = 5(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$$

$$\frac{50(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3})}{5(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})}$$

$$= 10(\cos(\frac{4\pi}{3} - \frac{\pi}{3}) + i \sin(\frac{4\pi}{3} - \frac{\pi}{3}))$$

$$\boxed{= 10(\cos \pi + i \sin \pi)}$$

$$10 \text{ cis } \pi$$

P.691 ex #7
Show how to get
Powers of complex
#s in polar form.

→ connects complex #'s & trig

DeMoivre's Theorem

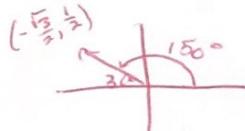
Let $z = r(\cos\theta + i \sin\theta)$ be a complex number in polar form. If n is a positive integer, then z to the n th power is:

$$z^n = [r(\cos\theta + i \sin\theta)]^n = r^n (\cos^{n\theta} + i \sin^{n\theta})$$

$$r^n (\cos(n\cdot\theta) + i \sin(n\cdot\theta))$$

5) Find $[2(\cos 30^\circ + i \sin 30^\circ)]^5$

$$= 2^5 [\cos(5 \cdot 30^\circ) + i \sin(5 \cdot 30^\circ)]$$



$$= 32 (\cos 150^\circ + i \sin 150^\circ)$$

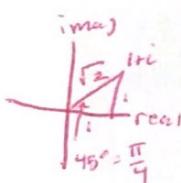
$$= 32 \left(-\frac{\sqrt{3}}{2} + i \frac{1}{2}\right)$$

$$= -16\sqrt{3} + 16i$$

$32 \text{ cis } 150^\circ$

or $32 \text{ cis } \frac{5\pi}{6}$

6) Find $(1+i)^4$ using DeMoivre's Theorem. Write the answer in rectangular form.



$$r = \sqrt{a^2 + b^2}$$

$$= \sqrt{1^2 + 1^2}$$

$$= \sqrt{2}$$

$$\tan \theta = \frac{1}{1}$$

$$\tan \theta = 1$$

$$\pi/4$$

$$(r \text{ cis } \theta)$$

$$1+i = r(\cos\theta + i \sin\theta) = \sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right) \quad \sqrt{2} \text{ cis } \frac{\pi}{4}$$

$$(1+i)^4 = (\sqrt{2})^4 [\cos(4 \cdot \frac{\pi}{4}) + i \sin(4 \cdot \frac{\pi}{4})]$$

$$= 4 (\cos \pi + i \sin \pi) \quad 4 \text{ cis } \pi$$

$$= 4(-1 + 0i)$$

$$= \boxed{-4} \text{ or } \boxed{-4+0i}$$